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ON A PROPERTY OF AN IMAGINARY LINE PASSING THROUGH ONE OF THE CIRCULAR POINTS AT INFINITY.

By Mr. James McMahon, Ithaca, N. Y.

In the *Messenger of Mathematics*, Vol. XVI. p. 2, appears the important statement that it would be more correct to say that the line $y = ix$ makes an indeterminate angle with itself, than to say (as is commonly done) that it is perpendicular to itself. The analytical reason given is that $\frac{m - m'}{1 + mm'}$ takes the form 0/0 when $m = m' = i$. This reason does not seem quite conclusive.

I think, for analytical purposes, it would be more satisfactory to state the the principle as follows:—

If two variable lines make any constant angle φ (real or imaginary) with each other, and if the equation of one of them approach the limiting form $y - y_0 = ix$, so will the equation of the other (unless $\tan^2 \varphi = -1$).

To prove this, let $m = (1 + \alpha)i$, and $m' = (1 + \alpha')i$;

then
$$\tan \varphi = \frac{\alpha - \alpha'}{\alpha + \alpha' + \alpha\alpha'} i, = k, \text{ suppose};$$

$$\therefore \frac{\alpha'}{\alpha} = \frac{i - k}{i + (1 + \alpha)k} = \frac{i - k}{i + k}, \text{ when } \alpha \doteq 0;$$

therefore, when k^2 is any constant except -1, α and $\alpha' = 0$ together, and the equation of each line approaches the limiting form stated above.

When a constant angle φ turns round a fixed point in a plane, its sides generate two homographic pencils, which also form a system in involution; and the above principle shows that when a ray of either pencil approaches the position in which it passes through either of the circular points at infinity, then will the corresponding ray of the other pencil approach the same position. These two limiting positions give the two double lines of the involution-system.

When $\tan^2 \varphi = -1$, the line that makes an angle φ with either of the lines $y = \pm ix$, may have any direction whatever.